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Segal's Mechanism for the Red Shift and Its Physical Bases

Jan Tarski

Scuola Internazionale Superiore di Studi Avanzati, 34014 Trieste, Italy

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Segal's mechanism for the red shift is reviewed, and is examined from several points of view. In particular, its compatibility with the expansion hypothesis is discussed. An appendix is devoted to some new considerations on cosmic numbers.

1. INTRODUCTION

Some ten years ago Segal suggested a new mechanism for the red shift (Segal, 1972, 1974). Segal, moreover, presupposed the Einstein static universe as background, and so he obtained a theory which constitutes an alternative to the expansion hypothesis. Since then this theory has been discussed (in print) by only very few scientists, even though it has a number of attractive features.

One of the goals of the present paper is to reinterpret Segal's theory. In particular, we assert that his mechanism is compatible with the expansion hypothesis. This mechanism may therefore modify the calculations based on the expansion hypothesis, without requiring that they be abandoned.

Another goal of this paper is to emphasize some foundational aspects of Segal's theory. In particular, this theory has suggested a new way of constructing quantized fields, as well as quantum states, on curved spaces. An examination of such fields and states, in turn, provides some theoretical support for Segal's mechanism. Moreover, this mechanism seems to suggest also some modifications or extensions in the theory of gravitation. (Segal's theory has led, in addition, to mathematical investigations of conformal-invariant wave equations; cf., e.g., Segal et al., 1981.)

In discussing Segal's mechanism, it is often very convenient to follow Segal and to assume the Einstein static universe $\tilde{M}$ as background, and we

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will usually do so. We recall that $\tilde{M} = R^1 \times S^3(\rho)$, where $S^3(\rho)$ is a three-sphere whose radius ρ is the radius of the universe. We summarize now some of the ingredients of this mechanism, and refer to Section 2 and to the references for further details.

The Minkowski-conformal group (rather, a covering of this group) has a natural action on $\tilde{M}$. The subgroup leaving a point $v \in \tilde{M}$ fixed is isomorphic to the Lorentz group with dilatations, and there is an associated relativistic time parameter v^0. Now, the space $\tilde{M}$ is conformally flat, and Maxwell's equations have conformal invariance. Consequently, after a light ray (or a photon) is emitted, the subsequent propagation preserves the ray's Minkowski-space characteristics. When the light ray is observed at $w \in \tilde{M}$, then the relativistic time w^0 at w enters into the picture. The scales and the directions of the two times differ, and roughly, this is the origin of the red shift.

We said "roughly" above, because there is some question about the roles of the two times v^0 and w^0. Indeed, one could say that the precise structure of the mechanism has not been fully clarified. We favor the view that the red shift occurs because the conformal invariance of equations for massless particles does not extend in a direct way to quantized fields (and the two times are conformally related).

However, some conclusions can be drawn which are independent of the details of the mechanism. The structure of $\tilde{M}$ shows that $\tilde{M}$ has a global or absolute time t. The relativistic time v^0 agrees with t at v (and similarly, w^0 agrees with t at w). In fact, there is a kind of tangency between relativistic and global parameters, and if a point v' is a distance D away from v, then at v', $t = v^0 + \mathcal{O}((D/\rho)^3)$. This implies that the dependence of the red shift $z(\tau)$ on a distance parameter τ is quadratic for small τ,

$$z(\tau) = (\text{const})(\tau/\rho)^2 + (\text{higher-order terms}) \tag{1}$$

We recall that $1 + z = \nu_{\text{em}}/\nu_{\text{obs}}$.

We can outline the empirical situation as follows. We select a reasonable value for ρ, and for small τ we may write

$$z(\tau) = B(\tau/\rho) + C(\tau/\rho)^2 + (\text{h.o.t.}) \tag{2}$$

If we suppose that $z(\tau)$ is due entirely to expansion, then we should have $|C| \ll B$ (Weinberg, 1977; Raine, 1981, pp. 36 and 37). This supposition is generally accepted, but it continues to be questioned by various authors (Abramenko, 1982; Narlikar, 1976; Nicoll and Segal, 1975, 1978; Segal, 1972–1978). In particular, an alternative analysis of data by Nicoll and Segal (1975, 1978) shows in effect that $|B| \ll C$. This conclusion neatly supports Segal's theory (on $\tilde{M}$). However, a permanently static universe seems to be an absurdity (Raine, 1981, p. 121). For example, it would have reached thermal equilibrium long, long ago. We therefore venture to say, that in (2) both B and C may be significant.

As we mentioned, Segal's mechanism can be reconciled with expansion, and the respective contributions will evidently be given by the C term and by the B term (cf. Section 3). The resulting combination has a somewhat *ad hoc* character, and does not show the elegance of Segal's mechanism when the latter is based on $\tilde{M}$. But such a superposition of effects should not be surprising. We may indeed replace (2) by the more general expression,

$$z(\tau) = A + B(\tau/\rho) + C(\tau/\rho)^2 + (\text{h.o.t.}) \tag{3}$$

where A is the gravitational red shift. Then, from our point of view, the three terms would have mutually different origins.

[The analysis of Nicoll and Segal (1975, 1978) could now be interpreted to mean that the present-day expansion, or contraction, is very small, without prejudice to expansion or contraction in the past or in the future.]

In the latter part of this paper we discuss two aspects of Segal's mechanism which were noted before. We argue in Section 4 that this mechanism is a very natural one, from a purely theoretic point of view. In Section 5 we consider some implications of this mechanism for the gravitational field.

We also take this opportunity and include an appendix which has only an indirect bearing on the main text. Namely, we discuss the cosmic numbers, in particular some relations involving ρ. We speculate that Planck's constant $\hbar$ may be related to ρ and to the totality of matter in the universe, through an extension of Mach's principle.

We should like to emphasize one basic shortcoming of this paper. Namely, we touch upon various technical questions, of a mathematical or physical nature, which have not been resolved. A large part of our discussion is therefore somewhat speculative. (As an example, cf. our previous comment about the roles of the two times v^0 and w^0.) However, some of these technical questions could take a long time to be cleared. We felt that it might be desirable to have an overall picture of Segal's mechanism and of its perspectives now.

2. SUMMARY OF SEGAL'S MECHANISM

One could say that Segal's theory originated in the analysis of the Minkowski-conformal group. In effect, Segal exploited constructions such as Penrose (1965, 1968) utilized for the study of the infinity of the Minkowski space. Such constructions can also be found in standard texts (Hawking and Ellis, 1973).

In these constructions the Minkowski space is imbedded in a space having the topological structure of $R^1 \times S^3$. Some further hypotheses about the latter space are necessary in order to carry out red shift calculations. We assume specifically $\tilde{M}$, as in the works of Segal and Penrose (and as in the Introduction). The axis of absolute time, i.e., the factor R^1, is to be determined by the condition that the average momentum with respect to this time axis should be zero.

The summary that follows is adapted from Segal (1974) and Tarski (1980a). The structure $\tilde{M} = R^1 \times S^3(\rho)$ implies a metric $\tilde{g}$ for $\tilde{M}$, and we use the signature $+\ -\ -\ -$. The metric $\tilde{g}$ is conformally flat, and this fact is crucial for the analysis which follows. In particular: Given $v \in \tilde{M}$, there exists a unique scalar function λ_v (defined on a neighborhood M_v of v) such that

$$g_v(w) = \lambda_v(w)\tilde{g}(w), \qquad w \in M_v \tag{4}$$

is a flat metric, and such that

$$\lambda_v(v) = 1, \qquad \lambda_v(w) > 1 \text{ for } w \neq v \tag{5}$$

The flat metric g_v can be correlated in a natural way with that of the tangent space $M_v^{\tan}$ at v, this latter space being a copy of the Minkowski space. Indeed, we can construct a unique map σ_v, the conformal map, which maps $M_v^{\tan}$ onto M_v, isometrically with reference to g_v,

$$\sigma_v : M_v^{\tan} \to M_v \subset \tilde{M} \tag{6}$$

This map also serves to define Minkowski-like parameters on M_v. The subset M_v is therefore that part of the universe $\tilde{M}$ which an observer at v can identify with the Minkowski space.

To proceed further, it is convenient to introduce in particular two parametrizations. We may designate a point of $\tilde{M}$ by $(t, u^1, \ldots, u^4)$ subject to $\Sigma(u^j)^2 = \rho^2$. Let $v = (0, \ldots, 0, -\rho)$. The Minkowski parameters $(v^0, \ldots, v^3)$ are then uniquely determined by the condition that for small values of $D := \max|v^\mu|$,

$$u^k = v^k + \mathcal{O}\big((D/\rho)^3\big), \qquad t = v^0 + \mathcal{O}\big((D/\rho)^3\big) \tag{7}$$

where $k = 1, 2, 3$. The explicit transformation formulas between the two sets of parameters can be found in Segal (1974) and Tarski (1980a). For reference we give one such formula,

$$t/\rho = \tan^{-1}\left[(v^0/\rho)(1 - V/4\rho^2)^{-1}\right], \qquad \text{where } V = (v^0)^2 - \Sigma(v^k)^2 \quad (8)$$

(We take $c = \hbar = 1$.) We also give the explicit form of λ_v (Hawking and Ellis, 1973),

$$\lambda_v(v^0,\ldots,v^3) = \left[1 + (v^0 + |\mathbf{v}|)^2/4\rho^2\right]\left[1 + (v^0 - |\mathbf{v}|)^2/4\rho^2\right] \quad (9)$$

where $|\mathbf{v}| = (\Sigma(v^k)^2)^{1/2}$.

We introduce the Hamiltonian operators

$$H^t = -i^{-1}\partial/\partial t, \qquad H_v = -i^{-1}\partial/\partial v^0 \quad (10)$$

One can show that

$$H^t > H_v \quad (11)$$

and since $H_v \geqslant 0$ for a state of free photons, then also $H^t > 0$. Moreover, $[H^t, H_v] \neq 0$. This relates, e.g., to the fact that the scale of v^0 is not uniform with respect to that of t.

We turn to the determination of the red shift. We follow Segal (1974) and set

$$H_v(\tau) = \exp(i\tau H^t)H_v\exp(-i\tau H^t) \quad (12)$$

Here τ is the time interval, measured along the t axis, over which the photon propagates. Note that τ is now a natural measure of distance between two points on the sphere $S^3(\rho)$ [cf. (1)].

Now, in Segal (1974) it was argued that a measurement of frequency corresponds to the scale-covariant parts of the energy, which are the energies H_v and $H_v(\tau)$, and whose expectations are therefore ν_{em} and ν_{obs}, respectively. An explicit determination of $H_v(\tau)$ and of its action on a (nearly) plane wave shows that $z(\tau) = \tan^2(\tau/2\rho)$. This evaluation then yields (1) with const $= 1/4$. This evaluation differs from that which would be obtained by a naive application of (8) with $V = 0$, namely, $z = \tan^2(\tau/\rho)$.

The scheme just outlined raises the following two questions, which are related. First: Since $\tilde{M}$ is static, the t energy (determined by H^t) should be conserved. How can this conservation be reconciled with the red-shift mechanism? An answer along the following line is given in Segal (1974). The emitted energy at v is the total t energy that is relevant (since near v, $H' \approx H_v$). For contrast, the absorbed energy at w is only a part of the relevant t energy, while the remainder, described by $H' - H_v(\tau)$, remains as free radiation.

Second: Can the red shift be derived in a more systematic way from Maxwell's equations adapted to $\tilde{M}$? In particular, from where comes the identification $\nu_{\text{obs}} = \langle H_v(\tau)\rangle$, and how should the (liberated) free radiation be described?

A preliminary study in this direction was attempted in (Tarski, 1977), in the approximation of scalar photons and in the framework of classical fields. The point of view there presented could be called a converse to that based on the operator $H_v(\tau)$ of (12). Namely, the basic premise of Tarski (1977) is that the interaction with sources (i.e., emission and absorption) is governed by the metric $\tilde{g}$, while the free propagation, in view of conformal invariance, combines the influence of both metrics $\tilde{g}$ and g_v. The relevant Hamiltonian at v would then be $H' \approx H_v$, and that at w would be $H' \approx H_w$, rather than $H_v(\tau)$. We see here a more symmetric approach than the previous one. Moreover, the state vector, or Green's function, would be "weakened" upon propagation from v to w.

This weakening can be handled more satisfactorily in the framework of quantized fields, which allow one to deal with changing numbers of particles. An analysis based on quantized fields is outlined in Tarski (1980a), again in the approximation of scalar photons. The basic underlying observation is the following: Translations, rotations, and Lorentz boosts induce transformations of quantized fields, which preserve particle number in case of free fields on Minkowski space. However, transformations of fields which are induced by conformal transformations change particle number. [This is why we said "not...in a direct way" in the introduction. But see Section 4, point (iv).] In particular, a photon state vector corresponding to one photon emitted at v will appear at w as a conformal-transformed state vector, the latter describing a red-shifted photon and a cloud of soft photons.

The crude approximations of Tarski (1980a) give the red shift

$$z \approx \lambda_v^{1/4} - 1 = \left[1 + (\tau/\rho)^2\right]^{1/4} - 1 \approx \tfrac{1}{4}(\tau/\rho)^2 \tag{13}$$

where we used (9) and $v^0 = |\mathbf{v}| = \tau$. The same applies to Tarski (1977), but cf. footnote in Tarski (1980a, p. 335). The value in (13) agrees for small τ with $\tan^2(\tau/2\rho)$ given before.

We should like to give some technical details. The usual approach to constructing quantized fields on a manifold such as $\tilde{M}$ is to consider fields directly as operator-valued distributions, which yield operators when multiplied by test functions and integrated over the manifold. Examples of such fields can be found in (Isham et al., 1981). Let us consider in particular the free scalar massless field χ on $\tilde{M}$, which is defined by the equations

$$\left(-\nabla^{\mu}\nabla_{\mu}+\tfrac{1}{6}R\right)\chi=0 \tag{14a}$$

$$[\partial_t\chi(u),\chi(u')]=(i^{-1})^3\tilde{\delta}(u,u') \tag{14b}$$

where in the commutator we assume the same time t of $\chi(u)$ and of $\chi(u')$. Moreover, R is the scalar curvature of $\tilde{M}$,

$$R=-6/\rho^2 \tag{15}$$

and ${}^3\tilde{\delta}$ is associated with the spatial part ${}^3\tilde{g}$ of $\tilde{g}$ in the usual way:

$$\int_{D\subset S^3(\rho)} d^3u|\det{}^3\tilde{g}(u)|^{1/2}({}^3\tilde{\delta})(u,u')=1 \text{ or } 0 \tag{16}$$

The construction of the Fock space for this field also follows in the usual way.

Such a field would not yield a red shift. Indeed, if we consider the Feynman diagrams for emission and for reabsorption of photons in the lowest order, then we would have conservation of t energy at each vertex, and hence no possibility of a red shift. In higher orders there would be some radiative energy loss, but this would be a qualitatively different phenomenon from red shift.

In order to describe the red shift, we can start with the free scalar massless fields $\varphi^{(0)}$ on the tangent spaces $M_v^{\tan}$, $M_w^{\tan}$, ... and then transfer these fields with the help of maps which are induced from $\sigma_v, \sigma_w, \ldots$. We will denote the resulting fields on $M_v, M_w, \ldots \subset \tilde{M}$ by $\varphi_v, \varphi_w, \ldots$.

Then, if a (scalar) photon is emitted at v, it should be described with the help of φ_v. However, to analyze the absorption process at w, we should use the transformed field $\varphi_w \sim U\varphi_v U^{-1}$. We refer to Tarski (1980a) for further details. We only note that it is intuitively to be expected, given nonconservation of particle number and a transformation that is near the identity, that a one-photon state should be transformed into a state containing a red-shifted photon and a cloud of soft photons. In this way the energy balance can be retained.

We note the following transformation rule: If suitable conventions are made, then

$$\varphi_w(w)\sim U\varphi_v(w)U^{-1}\sim\varphi_v(w)\lambda_v^{1/2}(w) \tag{17}$$

where we wrote $\sim$ in place of $=$, since the mathematical meaning of the transformation has not yet been made precise. The factor $\lambda_v^{1/2}$ gives the red shift in (13) and will also be of interest to us in another connection.

3. NONSTATIC BACKGROUND

We turn to the problem of reconciling Segal's mechanism with a nonstatic background, in particular, with a Robertson–Walker metric (Hawking and Ellis, 1973; Raine, 1981). Let du indicate the infinitesimal distance on $S^3(\rho)$, so that $ds^2 = dt^2 - du^2$ on $\tilde{M}$. We now consider a manifold $\tilde{M}'$ with

$$ds^2 = dt^2 - r^2(t)\,du^2 \tag{18}$$

i.e., $r(t)\rho$ is the radius of the universe at time t. For simplicity of discussion we assume that $r(t)$ is defined and continuous for all t.

We make a change of variables, $dt' = dt/r(t)$ and $r(t) = \alpha(t')$, and obtain

$$ds^2 = \alpha^2(t')(dt'^2 - du^2) \tag{19}$$

This metric is conformally flat, and we may proceed along the lines considered before. In particular, we may parametrize $\tilde{M}'$ by (t', u), and in this way adapt the formulas of Section 2 for λ_v, etc. to this space. However, before analyzing the red shift, one comment should be made.

In Section 2 we suggested describing the emission at v with the help of the metric $\tilde{g}(v)$, and then the free propagation should show the characteristics of both $\tilde{g}$ and $g_v = \lambda_v \tilde{g}$. In addition, we suggested using the field φ_v induced by the map σ_v from the tangent space. In the present case it is clearly natural to use the metric

$$g_v = \lambda_v \alpha^{-2} \tilde{g}_\alpha \tag{20}$$

and the corresponding map $\sigma_{v,\alpha}$. There is, however, the question, how should $\lambda_v \alpha^{-2}$ be characterized, in order to have a well-defined procedure? In other words, why use $\lambda_v \alpha^{-2}$ rather than some other $\lambda_{v'} \alpha^{-2}$? The conditions (5) defining λ_v are not fulfilled in general by $\lambda_v \alpha^{-2}$. But such conditions will be suitable if v and w are restricted to have the same t coordinate, $t_v = t_w$. Explicitly, we have the following: The conditions

$$\bar{\lambda}_v(v) = \alpha^{-2}(t'_v) \qquad \text{and} \qquad \bar{\lambda}_v(w) > \alpha^{-2}(t'_v) \tag{21}$$

(for all w such that $w \in M_v, w \neq v, t_w = t_v$), are fulfilled by $\bar{\lambda}_v = \lambda_v \alpha^{-2}$, and they determine $\bar{\lambda}_v$ uniquely. The map $\sigma_{v,\alpha}$ is then also determined.

We turn to the red shift. Since the precise structure of Segal's mechanism has not been fully clarified in case of $\tilde{M}$, we can only speculate about the form that it may take on $\tilde{M}'$. For example, the factor $r^2(t)$ may contribute to the red shift through a Doppler effect (cf. Raine, 1981), and the amount should then be compounded with the red shift as calculated before for $\tilde{M}$.

An alternative approach depends on applying the methods of Section 2 directly to $\tilde{M}'$ (and on combining with the Doppler effect, which we ignore here). The approximations of Tarski (1980a or 1977) then give, as in (13),

$$1 + z' \approx \left[\bar{\lambda}_v(w)/\bar{\lambda}_v(v)\right]^{1/4} = \lambda_v^{1/4}(w)\left[\alpha(t'_v)/\alpha(t'_w)\right]^{1/2} \qquad (22)$$

[The reason for the factor $\bar{\lambda}_v(v)$ is evident.] We wrote z' to indicate that this red shift refers to frequencies measured by t'. To convert to frequencies measured by t, we should include an additional factor $\alpha(t'_w)/\alpha(t'_v)$ and use $\alpha(t') = r(t)$. The result is

$$1 + z \approx \lambda_v^{1/4}(w)\left[r(t_v)/r(t_w)\right]^{1/2} \qquad (23)$$

If we set $t_v = 0$ and $t_w = \tau$, and assume that in the interval $[0, \tau]$,

$$|\dot{r}| \approx \text{const} \ll r(0)/\tau \qquad (24)$$

then we recover the form (2),

$$z \approx \tfrac{1}{2}(\dot{r}\rho)\left[\tau/r(0)\rho\right] + \tfrac{1}{4}\left[\tau/r(0)\rho\right]^2 + (\text{h.o.t.}) \qquad (25)$$

[One can replace here $r(0)$ by $r(\tau)$ if desired.] In this latter approach, the expansion factor $\dot{r}(t)$ contributes to the red shift through both Segal's mechanism and the Doppler effect.

4. DISCUSSION OF THE MECHANISM

We assume now an arbitrary conformally flat background space, together with its perturbations. However, for definiteness we will refer to $\tilde{M}$. We should like to offer some general theoretical considerations for using the flat-space constructs like φ_v, rather than global structures like χ. This amounts to giving theoretical arguments for Segal's mechanism.

(i) *The picture of evolution and a principle of equivalence.* The quantum-field-theoretic scheme that we outlined can be portrayed as follows. When a photon (or a scalar quantum) is first emitted, it "senses" a flat neighborhood. As it moves, its relation to its flat neighborhood changes, and the curvature has its specific effects. In particular, a one-photon state vector can evolve to a many-photon state vector. But the entire process is constructed from elementary vectors which are associated with a flat space.

One could also say that a picture of this kind is suggested by the general-relativistic principle of equivalence, if the latter is extended to the quantum domain.

(ii) *Perturbations of* $\tilde{M}$. An additional argument for the foregoing picture of evolution comes from a consideration of perturbations of $\tilde{M}$. We recall that the construction of a photon state (quantum-mechanical, with $s = 1$) is a rather nontrivial procedure, and is one which requires the detailed structure of the Poincaré group, or, e.g., of twistors. This construction can be carried out in a conformal-invariant way, and then it can be taken directly from the Minkowski space to $\tilde{M}$.

However, if we want to deal with quantum states directly over manifolds, then we have to take into account also perturbations of $\tilde{M}$ which in general are not conformally flat. (This point should be relevant, if we are interested in discussing photon states over large distances.) Now, it seems a reasonable hypothesis, that a direct construction of a photon state over a non-conformally-flat space is not possible. The difficulties which have been encountered with non-null hypersurface twistors (Penrose and Ward, 1980) indeed would favor this hypothesis. On the other hand, the construction by transfer from flat spaces requires that we find an appropriate generalization of the map σ_v of (6) to non-conformally flat spaces. This is perhaps a more manageable task.

We emphasize that the (conjectured) impossibility of a direct construction applies specifically to massless particles with spin. For particles with mass or without spin, constructions directly on manifolds are possible (cf., e.g., Isham et al., 1981). But it is a reasonable expectation, that all elementary particles on manifolds should be treated in the same way. [Then the studies of quantizing particles on manifolds, e.g., by path-integral methods, should be regarded as conceptually inappropriate. Cf., e.g., Tarski (1982) and references given therein.]

(iii) *Locality.* Considerations of locality similarly suggest the use of fields like φ_v and the foregoing picture of evolution, in preference to global fields like χ. The fields φ_v can be constructed locally, while for χ, we would have to have information about the global structure of space, and to take into account any singularities. A local description harmonizes better with general precepts.

(iv) *Conformal transformations leaving the vacuum invariant.* For completeness we repeat the following possibility, which was mentioned in Tarski (1980a) but which was rejected there as inappropriate for handling the red shift problem. We recall that conformal transformations were constructed and analyzed which would allow the use of fields like φ_v without a red shift (Hortaçsu et al., 1972; Swieca et al., 1973). These transformations depend on separating a free field into its creation and annihilation parts, and then not mixing the respective parts. But the separation in question is a highly nonlocal operation. Furthermore, such a procedure would bypass the Lagrangian aspects of the theory, and it is not clear how it could be incorporated into a consistent scheme of interactions.

5. IMPLICATIONS FOR THE GRAVITATIONAL FIELD

Segal's mechanism and the accompanying point of view raise some new problems for the quantum-theoretic gravitational interactions. The physically interesting interactions include especially those of the electromagnetic field and of gravitational waves. We will, however, continue to use the scalar massless fields for examples.

For simplicity we take again the Einstein static universe $\tilde{M}$ as background, and we will allow small perturbations. We recall (Hawking and Ellis, 1973; Raine, 1981) that $\tilde{M}$ constitutes a solution to the equation

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda - 8\pi G T_{\mu\nu}^{(0)} = 0 \tag{26}$$

where $T_{\mu\nu}^{(0)}$ has the form

$$T_{00}^{(0)} =: \mu^{(0)} > 0, \qquad T_{\mu\nu}^{(0)} = 0 \text{ otherwise} \tag{27}$$

$\mu^{(0)}$ being a constant. By noting that in the coordinates (t, u) the spatial and the temporal components do not couple, and that $R_{00} = 0$ and $R = -6/\rho^2$, and by taking the three-trace and the four-trace, we obtain (cf. Raine, 1981, p. 121)

$$\Lambda = 4\pi G \mu^{(0)} = \rho^{-2} \tag{28}$$

For a perturbed space we assume

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda - 8\pi G T_{\mu\nu}^{(0)} = 8\pi G T_{\mu\nu} \tag{29}$$

where $T_{\mu\nu}$ is (in some sense) small.

We suppose now that we have a massless scalar field φ on $\tilde{M}$. We saw how the metrical structure of $\tilde{M}$ affects φ and its representations. We would then like to ask, how could the converse effect be determined.

Equation (29) clearly provides a means of describing the influence of φ and of its dynamic evolution on the metric. However, if φ interacts with a source field ψ, then the use of different representations of φ for the $\varphi-\psi$ interaction (i.e., the use of φ_v, φ_w, etc.) is affected by the curvature. So we expect to have a back-reaction: the effect of changes of representations on the metrical structure.

Now, two fields φ_v, φ_w will differ in general by factors $\lambda_v^{1/2}, \lambda_w^{1/2}$, etc.; cf. (14). Therefore we expect to have the factors $\lambda_v^{1/2}$ in $T_{\mu\nu}$. Such factors may have macroscopic consequences (even if not easily observable ones); cf. examples in Tarski (1980b) and Berg and Tarski (1982).

A more basic question that arises is that of conformal flatness and of conformal factors. We recall the problem mentioned in Section 4, that of generalizing the map σ_v to non-conformally-flat spaces. It appears that this problem should be regarded as in the domain of theory of gravitation. [We may also remark that the interplay of the metrics $\tilde{g}, g_v$ brings to mind the two-metric theories of gravitation. Such theories are briefly reviewed in Will (1979).]

APPENDIX. THE COSMIC NUMBERS AND MACH'S PRINCIPLE

The cosmic numbers constitute a recurrent theme of discussion. We have in mind of course (1) the number N of nucleons in the universe, each having the approximate mass m, so that $M := mN$ is (roughly) the total mass, (2) the gravitational constant G, and (3) the age T of the universe, which we suppose to be of the same order as its radius ρ. In atomic units ($c = \hbar = m = 1$), one has the familiar estimates

$$M = N \sim 10^{80}, \qquad \rho \sim 10^{40}, \qquad G \sim 10^{-40} \tag{A1}$$

The challenge now is to interpret the evident relations among these values. Cf. Carr and Rees (1979) for a recent discussion, from the anthropic point of view.

One such relation, which presupposes $c = 1$ but which depends neither on the value of $\hbar$, nor on m nor on N separately, is

$$MG \sim \rho \tag{A2}$$

This relation has been derived or otherwise discussed, in connection with Mach's principle and otherwise. See Sciama (1953) and Tarski (1980b) and references cited therein. One might say that here a preliminary orientation has been achieved. [Note that this relation is also implicit in (28).]

The quantities in (A1) admit one other independent relation, which would necessarily require atomic units. We consider in particular the following:

$$N \sim \rho^2 \tag{A3}$$

We do not know of any published discussion of this estimate in the context of cosmic numbers (but see below). Clearly, several kinds of interpretations can be envisaged. For example, one might try to interpret (A3) in terms of thermal equilibrium, the empty static or nearly static space having a definite associated energy.

We should like to offer the suggestion, that (A3) could be interpreted in terms of a dependence of $\hbar$ on N and ρ. Indeed, we regard such a dependence as stemming from a quantum-theoretic extension of Mach's principle. We recall the discussion of Mach (1960), where it is argued that a law of motion, in particular the law of inertia, is meaningful only in so far as motion can be related to other bodies, e.g., to the totality of matter in the universe. It is then natural to suppose that the same should apply to quantum mechanical motion, characterized by $\hbar$.

We can give an example in which $\hbar$ is considered as a variable. Let us introduce another value, $\hbar_1$, defined by the condition that the universe be close-packed with nucleons. For clarity of meaning we retain m in the formulas, and we set $\rho_1 := \hbar_1/m$, the new nucleon Compton wavelength. Close packing means that

$$\rho^3 \sim N\rho_1^3 \qquad \text{or} \qquad \rho \sim N^{1/3}(\hbar_1/m) \tag{A4}$$

while (A3) implies $\rho \sim N^{1/2}(\hbar/m)$. Therefore

$$\hbar \sim \hbar_1 N^{-1/6} \tag{A5a}$$

We can also put this into a form which has some resemblance to formulas for fluctuations in statistical physics:

$$\langle \Delta x \rangle \langle \Delta p_x \rangle \gtrsim \hbar_1 N^{-1/6} \tag{A5b}$$

It would be intriguing to construct a model theory which would illustrate how the combination $\hbar_1 N^{-1/6}$ might arise.

There is another interesting aspect to (A3). This relation is reminiscent of an analysis of Eddington (1953), carried out in the framework of classical physics. He argued that, given N particles in a three-sphere of radius ρ, and a coordinate system, then the origin of this coordinate system has an intrinsic fluctuation, whose standard deviation σ is given by

$$\sigma = 2\rho N^{-1/2} \sim \rho N^{-1/2} \tag{A6}$$

Let us combine this result with (A3). We obtain

$$\sigma \sim 1 \sim \hbar / m \tag{A7}$$

i.e., the standard deviation is roughly equal to the nucleon Compton wavelength. This is sensible on physical grounds. But we can take also the following point of view. If $c = m = 1$, then $\hbar$ is in effect a measure of length. Then $\hbar \sim \rho N^{-1/2}$, so that $\hbar$ is given directly in terms of cosmic quantities. This is in line with the extended Mach's principle, as suggested above.

We find these observations rather remarkable. However, it is not at all clear, how the classical fluctuations in Eddington's analysis might relate to $\hbar$ and to quantum-mechanical uncertainties.

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